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OBSERVATIONS OF EXOPLANETS TRANSITS IN THE FAR INFRARED WITH SPICA—SAFARI

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ABSTRACT

When the disks of a star and of a planet overlap as a consequence of the orbital motion, the total light coming from the system dims. If an instrument is sensitive enough to detect this flux variation, important parameters of the planetary system can be derived from the observed light curve. In this paper I show under which conditions the transits can be detected given the sensitivity of SAFARI.

Key words: Stars: planetary systems – Missions: SPICA – Techniques: photometric

1. INTRODUCTION

If the orbital plane of a planet is seen almost edge-on, during the transit of the planet in front of its star the received flux changes in time: the shape of the light curve depends on star's and planet's parameters that can be derived. The technique of the transit has been first used in the optical band but observations with the Spitzer Space Telescope at 24 μm (Richardson et al., 2006) have shown that it can be used also in the far infrared. At these wavelengths the flux from the star is greatly reduced, but modelling the transit is easier than in the visible where the limb darkening complicates the shape of the light curve (e.g. Mandel & Agol, 2002). At larger wavelengths this effect is strongly reduced and the light curve has a boxlike shape which allows a simple and robust fitting of the data.

In this work I derive under which conditions the transit of a planet can be seen in the SAFARI bands given the current estimates of the instrument sensitivity. In Section 2 I briefly give the basic equations that describe the geometry of the transit; in Section 3 I show the impact of the instrument sensitivity on the transit detection; the results are applied to a planetary system similar to ours in Section 4 and to other systems in Section 5; conclusions are given in Section 6.

2. THE GEOMETRY OF THE TRANSIT

Consider a planet of radius R_p orbiting in a circular orbit of radius a around a star of radius R_* . The disk of the planet intercepts the stellar surface if the system is seen

under an angle ι such that

$$\iota > \iota_2 = \arccos\left(\frac{R_* + R_p}{a}\right). \quad (1)$$

For a complete overlap of the planet's disk on the stellar surface the condition on ι is

$$\iota \geq \iota_1 = \arccos\left(\frac{R_* - R_p}{a}\right) > \iota_2. \quad (2)$$

When $R_p \ll R_*$ then $\iota_1 \simeq \iota_2$; for instance, the two angles for the Earth-Sun system are $\iota_2 = 89^\circ 73' (89^\circ 43' 54'')$ and $\iota_1 = 89^\circ 74' (89^\circ 44' 12'')$. For this reason in the following only the condition $\iota > \iota_1$ is considered.

If P is the orbital period, the duration of the transit is $P\varphi/\pi$ where φ is

$$\cos \varphi_{\pm} = \frac{1}{a \sin \iota} \sqrt{a^2 - (R_* \pm R_p)^2}$$

φ_+ gives the duration of the whole transit, while φ_- gives the interval during which the planet's disk is completely superimposed on the star's disk.

The projected distance d between planet and star varies with time according to

$$d = a \sqrt{\sin^2\left(\frac{2\pi t}{P}\right) + \cos^2 \iota \cos^2\left(\frac{2\pi t}{P}\right)}$$

where $t = 0$ is arbitrarily chosen to correspond to a minimum in d .

During the primary transit, when the planet is in front of the star, the stellar flux decreases by a fraction $f_* = \mathcal{S}/\pi R_*^2$ where \mathcal{S} is the overlapped area given by

$$\mathcal{S} = \begin{cases} 0 & \delta > 1 + p \\ R_*^2 \left[\arccos\left(\frac{\delta^2 + 1 - p^2}{2\delta}\right) - \frac{\rho}{2} + \right. \\ \left. + p^2 \arccos\left(\frac{\delta^2 + p^2 - 1}{2\delta p}\right) \right] & 1 - p \leq \delta \leq 1 + p \\ \pi R_p^2 & \delta < 1 - p. \end{cases} \quad (3)$$

$\delta \equiv d/R_*$ and $p \equiv R_p/R_*$ are adimensional variables and $\rho \equiv \sqrt{4\delta^2 - (1 + \delta^2 - p^2)^2}$.

During the secondary transit, when the planet is eclipsed by the star, the planet flux decreases by a fraction $f_p = \mathcal{S}/\pi R_p^2$.

From Equations (3) then $0 \leq f_* \leq p^2$ and $0 \leq f_p \leq 1$.

3. THE LIGHT CURVE OF THE TRANSIT

The following assumptions are made to simulate the light curve during the primary/secondary transit:

- the star emits like a blackbody with radius and temperature corresponding to its spectral type;
- the planet emits like a blackbody too, with a temperature of 126.8 K for Jupiter (Hildebrand et al., 1985) and 59.5 K for Neptune (Burgdorf et al., 2003);
- an observation consists of many 1 minute integrations: the noise n_λ associated to each integration is assumed to be pure Gaussian noise whose σ corresponds to the expected SAFARI sensitivities (see Table 1). Long term drifts (for instance 1/f noise) are not taken into account;
- in principle for a staring observation the cosmological confusion limit c_λ should be simply an (unknown) constant. However a certain amount of tracking inaccuracy is unavoidable, which translates into a jiggling of the telescope around the pointed position; in turn this means a temporal variation due to the spatial variation of c_λ . This effect is not modelled here and a constant term c_λ is simply summed to give a rough idea of the amplitude of this potential noise. c_λ 's are also reported in Table 1.

Table 1. The sensitivities of SAFARI. The first values are nominal, values in parenthesis are goal sensitivities. In the fourth column the confusion limit c_λ is reported, estimated from Grupioni & Pozzi (2009).

Effective λ (μm)	Sensitivities in μJy		Confusion limit in μJy
	5σ , 1 hour	1σ , 1 min	
48	23(11)	36(17)	8.7
85	34(13)	53(20)	260
160	58(18)	90(28)	6900

With these assumptions the observed flux can then be written as

$$F_o = (1 - f_\star)F_\star + (1 - f_p)F_p + n_\lambda + c_\lambda$$

The difference between the total flux outside the transit $F_o = F_\star + F_p + n_\lambda + c_\lambda$ and the minimum flux during the primary transit $F_o = (1 - p^2)F_\star + F_p + n_\lambda + c_\lambda$ is then $p^2F_\star + n_\lambda$. Clearly it is necessary that $p^2F_\star > n_\lambda$ to observe the light dim. For a given set (R_\star, T_\star, R_p) , the sensitivities reported in Table 1 can be turned into the largest distance D at which the planetary system can be observed with our instrument:

$$p^2F_\star = \frac{R_p^2}{R_\star^2} \frac{\pi R_\star^2}{D^2} \text{BB}(\lambda, T_\star) > n_\lambda$$

from which

$$D < R_p \sqrt{\frac{\pi \text{BB}}{n_\lambda}} \quad (4)$$

For any reasonable T_\star , a blackbody at SAFARI wavelengths is in the Rayleigh-Jeans regime so that the above equation can be rewritten as

$$D < 19.2 \frac{1}{\lambda(\mu\text{m})} \sqrt{\frac{T_\star(\text{K})}{n_\lambda(\mu\text{Jy})}} \left(\frac{R_p}{R_{\text{Earth}}} \right) \text{pc}$$

During the secondary transit the minimum flux is $F_o = F_\star + n_\lambda + c_\lambda$, so the difference is $F_p + n_\lambda$. Equation (4) is still valid but BB is now the planetary flux and the Rayleigh-Jeans approximation is no longer applicable.

Note that Equation (4) contains the planet's radius and not the ratio p . This means that there is no advantage in observing late type instead of early type stars on the base that the former have larger p . As we shall see, there is however a reason to consider late type stars: at short distance, less than 30 pc, the number of early type stars is very small.

4. TRANSITS AROUND SUN-LIKE STARS

4.1. EARTH-LIKE PLANETS

The primary transit of a planet like our Earth around a Sun-like star can be observed at distances $D < 7.5$ pc at $\lambda = 48 \mu\text{m}$ and assuming $n_\lambda = 17 \mu\text{Jy}$ (the goal sensitivity). The volume density of stars having $M_V = 4.5$ is 0.0023 pc^{-3} (Reid et al., 2002) which gives 1 star within 8 pc. In order to have a reasonable sample of G0 stars, say 100, we should reach 35 pc which, from Equation (4) solved for n_λ , implies a sensitivity better than $0.7 \mu\text{Jy}$!

4.2. OBSERVING GIANT PLANETS

For a given distance, Equation (4) can be inverted to find the minimum planet radius that can be observed with a given sensitivity. In Table 2 the results are given using both the nominal and the goal sensitivities shown in Table 1, for a solar star at 35 pc.

Table 2. The minimum planetary radius that produces a detectable transit around a $T = 6000 \text{ K}$ star at $D = 35 \text{ pc}$ (from Equation (4) adopting both nominal and goal sensitivities of Table 1).

Effective λ (μm)	Planet radius		
	Nominal	Goal	
48	6.8	4.6	Earth radii
85	1.3	0.8	Jupiter radii
160	3.3	1.8	Jupiter radii

As an example of light curve, in Figure 1 the primary transit of a planet with $R_p = 4.6 R_{\text{Earth}} \sim 1.2 R_{\text{Neptune}}$ at $48 \mu\text{m}$ is shown. The light dimming is clearly visible.

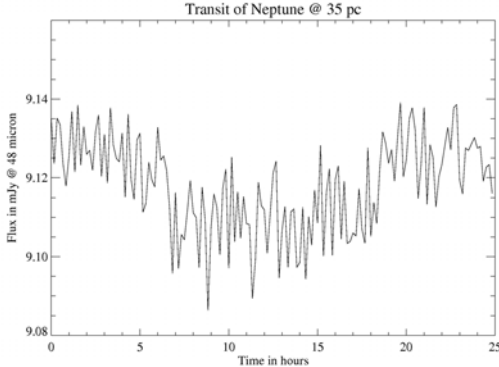


Figure 1. The transit of a planet with radius $1.2 R_{\text{Neptune}}$ orbiting in one year around a 6000 K star at 35 pc . The inclination of the orbital plane is $89^\circ.9$. The observation is simulated with elementary integrations of 1 minute which are then rebinned at 10 minutes. The orbital period determines the length of the transit, but it does not affect its depth.

The light curve of a planet with $R_p \sim 1.8 R_{\text{Jupiter}}$ at $160 \mu\text{m}$ is shown in Figure 2. Note that the DC level is dominated by c_λ , the star contributing with 0.8 mJy only. This implies that even a moderate telescope jiggling may destroy the transit visibility.

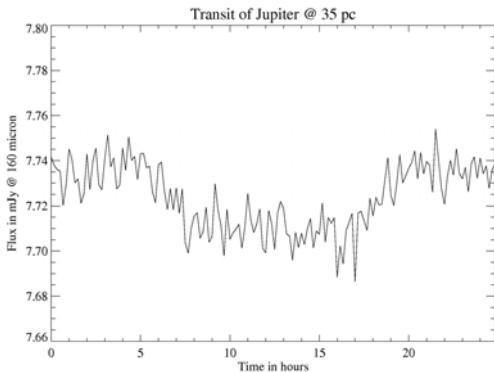


Figure 2. The transit of a planet with radius $1.8 R_{\text{Jupiter}}$ (see Figure 1 for details).

For what concerns the secondary transit, Equation (4) gives $R_p = 6 R_{\text{Jupiter}}$ for a temperature of 126.8 K and $n_\lambda = 17 \mu\text{m}$, which is almost three times the largest known planet radius ($2.2 R_{\text{Jupiter}}$).

5. OTHER PLANETARY SYSTEMS

SAFARI is not aimed to survey the sky looking for new planetary transits, but to execute pointed observations

around known targets. Thus, in order to derive what can be achieved with SAFARI, rather than exploring blindly the parameters space that describe the transits, it is more appropriate to consider only a smaller portion of this space based on the properties of the already known planetary systems, even if the current knowledge could, and certainly will, evolve in the future.

The largest planet radius observed with any of the available detection techniques is¹ $2.2 R_{\text{Jupiter}} (\sim 24 R_{\text{Earth}})$. Table 3 reports the largest distance, from Equation (4), at which such a planet can be observed: for the green and red band, c_λ has been used instead of n_λ .

Table 3. Largest distance at which a planet of radius $2.2 R_{\text{Jupiter}}$ can be observed as function of the spectral type of the hosting star (from Equation (4) adopting the nominal sensitivity at $48 \mu\text{m}$ and the confusion limit for the other two bands).

SpT	Distance in pc		
	(48 μm)	(85 μm)	(160 μm)
M3	94	20	2
K5	106	22	2
K0	116	24	3
G0	124	26	3
F5	128	27	3

It is not possible to observe the transit in the red band unless the pointing accuracy during the observation is such that the confusion limit plays no role. In the green band the main limitation is given by the sampled volume: the number of stars within 20 pc from the sun varies from ~ 60 for M3 stars to less than 10 for F5 (from Reid et al., 2002). Indeed the two stars with observed transit at a distance less than 20 pc are GJ 436 ($\sim \text{M3}$) and HD 189733 ($\sim \text{K0}$).

In Table 3 the dependence of the distance on the spectral type is not significant because in Equation (4) the spectral type enters only through the square root of the effective temperature which varies by a factor 2 from M3 to F5. Since late type stars are more numerous than early stars one may conclude that the number of known planetary systems should anticorrelate with the spectral type.

This conclusion is proved wrong by looking at Figure 3 which shows the mass/radius relation for the stars with planetary transit observed: the most abundant stars of late type M are less numerous than stars as early as F. This is due to: a) Equation (1) which favours a large ratio R_*/a ; b) an obvious constraint on the duration of the transit which favours short orbital periods: this in turn, assuming applicable the third Kepler's law, implies a small a^3/M_* . Both conditions make it easier the detection around early type stars.

¹ The source for the planetary data quoted in this section is <http://exoplanet.eu>.

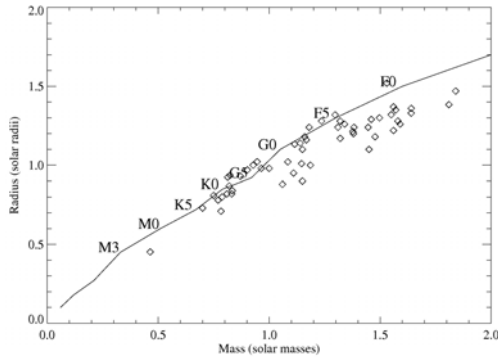


Figure 3. Solid line: mass/radius relation for main sequence stars (from Lang, 1992); rhombs: stars with observed planetary transit (from <http://exoplanet.eu/>).

This bias can be easily confirmed: if we consider all the planetary detections, 172 stars out of 229 (75%) have $R_* \leq 1R_\odot$, late type stars are more numerous, and 97 out of 311 planets (31%) have $a < 1$ AU, no clear preference for small a ; if we now consider only the exoplanets for which the transit is observed we find only 20 out of 57 (35%) stars with $R_* \leq 1R_\odot$ and 58 out of 60 (97%) with $a < 0.1$ AU, with $a_{\max} = 0.45$ AU.

Note that while the condition on short orbital periods is a pure observational constraint and may be relaxed in the future (for instance a mission like PLATO² does not have such a limitation), the condition on high inclination angle for transit detection can not be removed at all, causing a bias toward large R_*/a .

For what concerns the secondary transit, Equation (4) gives $D < 13$ pc for a $2.2 R_{\text{Jupiter}}$ observed at $48 \mu\text{m}$. An example of light curve is shown in Figure 4 for a M0 star at 10 pc.

6. CONCLUSIONS

In this paper I generated synthetic light curves of exoplanets transits to derive under which conditions they can be observed with SAFARI.

The results are here summarized:

1. The most favourable band is at $48 \mu\text{m}$.
2. The red band is not suitable if the noise is dominated by the confusion limit. This conclusion depends strongly on the pointing accuracy of the telescope, and on the theoretical model adopted to estimate the confusion limit. As such, it may be too pessimistic.
3. A similar conclusion holds for the green band, but less stringent and with the same caveat as before. In any

² PLATO (PLANetary Transits and Oscillations of stars) is one of the four medium class missions selected for assessment study in the framework of the ESA Cosmic Vision 2015-2025 program. See for instance <http://www.oact.inaf.it/plato/PPLC/Home.html>.

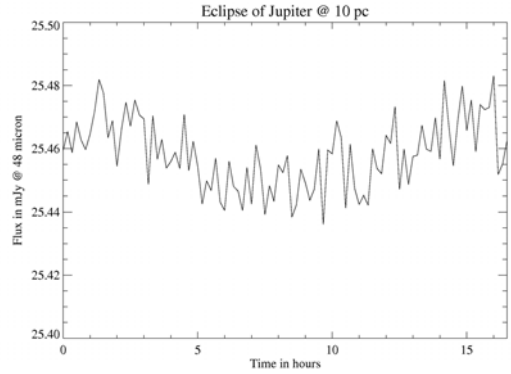


Figure 4. The secondary transit of a planet with radius $2.2 R_{\text{Jupiter}}$ ($T_{\text{eff}} = 126.8$ K) orbiting in one year around a M0 star ($R = 0.60 R_*$, $T_{\text{eff}} = 3850$ K) at 10 pc. The inclination of the orbital plane is $89^\circ.93$. For details on the light curve see Figure 1.

case unless in the far infrared the planet atmosphere can play a role in modifying the transit appearance, there is no clear advantage to observe in more than one band.

4. The observation of an Earth-like planet is out of the SAFARI possibilities, but the transit of a planet as small as Neptune can be observed. For very short distances, less than 10 pc, the eclipse (secondary transit) of a *double* Jupiter can be observed.
5. The variation of the flux in and out of transit depends on the planet radius and on the star temperature, but not on the star radius. The earlier the star spectral type is, the farther the transit can be observed.
6. An observational bias such that early type stars are preferred is unavoidable due to the constraint on the inclination angle; this bias overwhelms the fact that late type stars are more numerous.

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